THE KING'S SCHOOL, CANTERBURY

SCHOLARSHIP ENTRANCE EXAMINATION

March 2010

MATHEMATICS Paper I

Time: 45 minutes (plus reading time)

Use the reading time wisely; gain an overview of the paper and start to think of how you will answer the questions.

Do as many questions as you can (clearly numbered) on the lined paper provided.

The questions are not of equal length or mark allocation. Make sure you avoid spending too much time on any one question; don't get bogged down! Move on quickly if you get stuck. The paper is quite long; you are not necessarily expected to finish everything.

Some of the later questions are more difficult, but not necessarily longer. Some questions are designed to test your ability to work with unfamiliar ideas, or familiar ones with a twist. Don't give up!

*You are expected to use a calculator where appropriate, but also you must show **full and clear working**, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers. In fact, merely writing down an answer might score few marks.*

Complete questions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.

This paper has eleven questions and eight pages (last one blank).

1 We can create the n^{th} **Farey Sequence**, F_n , by writing down all the fractions between 0 and 1 (inclusive), **in order of size**, and **in lowest terms**, with denominators up to and including n.

The first Farey sequence, F_1 , is:

$$\frac{0}{1} \quad \frac{1}{1}$$

The second Farey sequence, F_2 , is:

$$\frac{0}{1} \quad \frac{1}{2} \quad \frac{1}{1}$$

The third Farey sequence, F_3 , is:

$$\frac{0}{1} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{1}$$

(a) Write out the fourth Farey sequence, F_4 , in order.

(b) Similarly, write down F_5 .

2 DJ Destroyer wants to play some dubstep music at a party.

He wants the music to have a tempo between 132 and 144 bpm (beats per minute).

He needs to know how many milliseconds (thousands of a second) there are between each beat.

He knows a formula which can help him, namely:

$$T = \frac{60000}{b}$$

where b is the bpm and T is the time in milliseconds.

Work out the minimum **and** maximum number of milliseconds DJ Destroyer can have between beats for the music to stay within the tempo range he wants. Give your answers as accurately as you can.

3 According to a national newspaper last year:

"2.4 million workers called in sick on the first Monday after New Year 2009. This was 8.3% of the British workforce."

Based on this figure, how big is the British workforce (show your working carefully and give your answer to an appropriate degree of accuracy)?

4

(a) Simplify as far as you can: $\dfrac{16x^4y^3}{56x^2y^2}$

(b) Factorise $42x^2 - 14x$

5

(a) The new high speed train service between Canterbury and central London now has a fastest service of 55 minutes. What percentage reduction is this from the original journey duration of 1 hour 40 minutes?

(b) Matt used to pay £22.40 for a ticket on the slow train and now reckons it costs 30% more to travel on the fast train. If this is true, work out the price of the fast train ticket (to the nearest penny).

(c) Heather claims that the new fast trains mean her usual journey on a slow train now takes 20% longer at 1 hour 48 minutes (owing to it having to make way for the fast trains).

What was the duration of her journey originally?

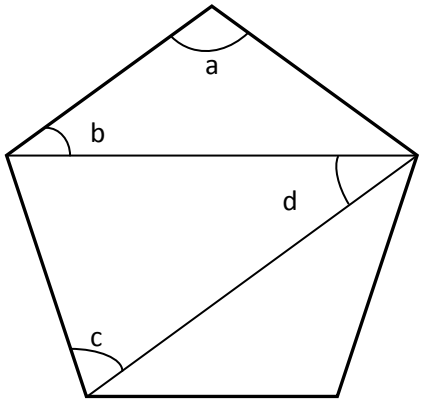
6 Solve the following equations:

(a) $3(x-4) = -57$

(b) $\dfrac{4x}{5} + 3 = 31$

(c) $4 - 3(5-x) = x + \frac{1}{2}(x-4)$

7 Work out each of the lettered angles in the **regular** pentagon below. Show any necessary working.



8 Write 2010 as a product of its prime factors.

9 Kristin performs a show at the Gulbenkian Theatre in Canterbury.

There are 340 seats available for the show.

Tickets are £13 for adults and £7 for children. Total tickets sales are £3790.

There are *x* adult tickets sold and *y* children's tickets. All the seats are sold.

(a) Explain clearly why the above information gives rise to the following pair of simultaneous equations:

$$x + y = 340$$

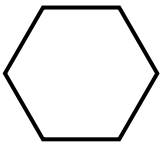
$$13x + 7y = 3790$$

(b) Solve these equations.

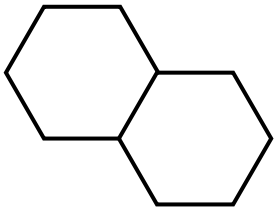
(c) The theatre pays Kristin £650 plus half the money from ticket sales. How much does she receive in total?

10 Consider the following sequence of shapes made from hexagons and answer the questions below.

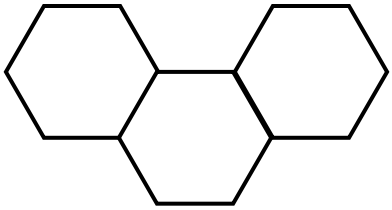
n=1



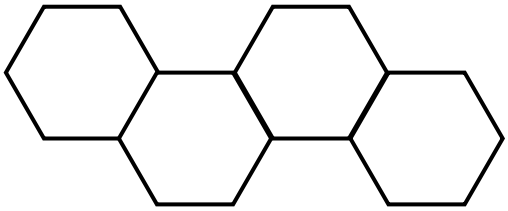
n=2



n=3



n=4



***n* (number of hexagons)**	***E* (number of edge lines)**
1	6
2	11
3	16
4	21

(a) How many edge lines will there be with five hexagons?

(b) Find a formula for E in terms of n.

(c) A pattern has 10051 edges lines; how many hexagons does it contain?

11 This question is about GCSE examinations taken by 16-year olds in the UK.

It uses the table of data given opposite and is ordered by the percentage of students achieving grade C or better.

Some subjects (mathematics and English) are compulsory at GCSE. Other subjects are optional.

Some students study combined science but the better scientists study separate sciences.

Write careful comments on each of the following statements. Say whether you agree or disagree with the statements and give clear reasons as to why.

(a) Alf says: "classical studies is the easiest subject – look at how many A* are achieved".

(b) Billie says: "mathematics is the hardest because a greater proportion achieves less than a C."

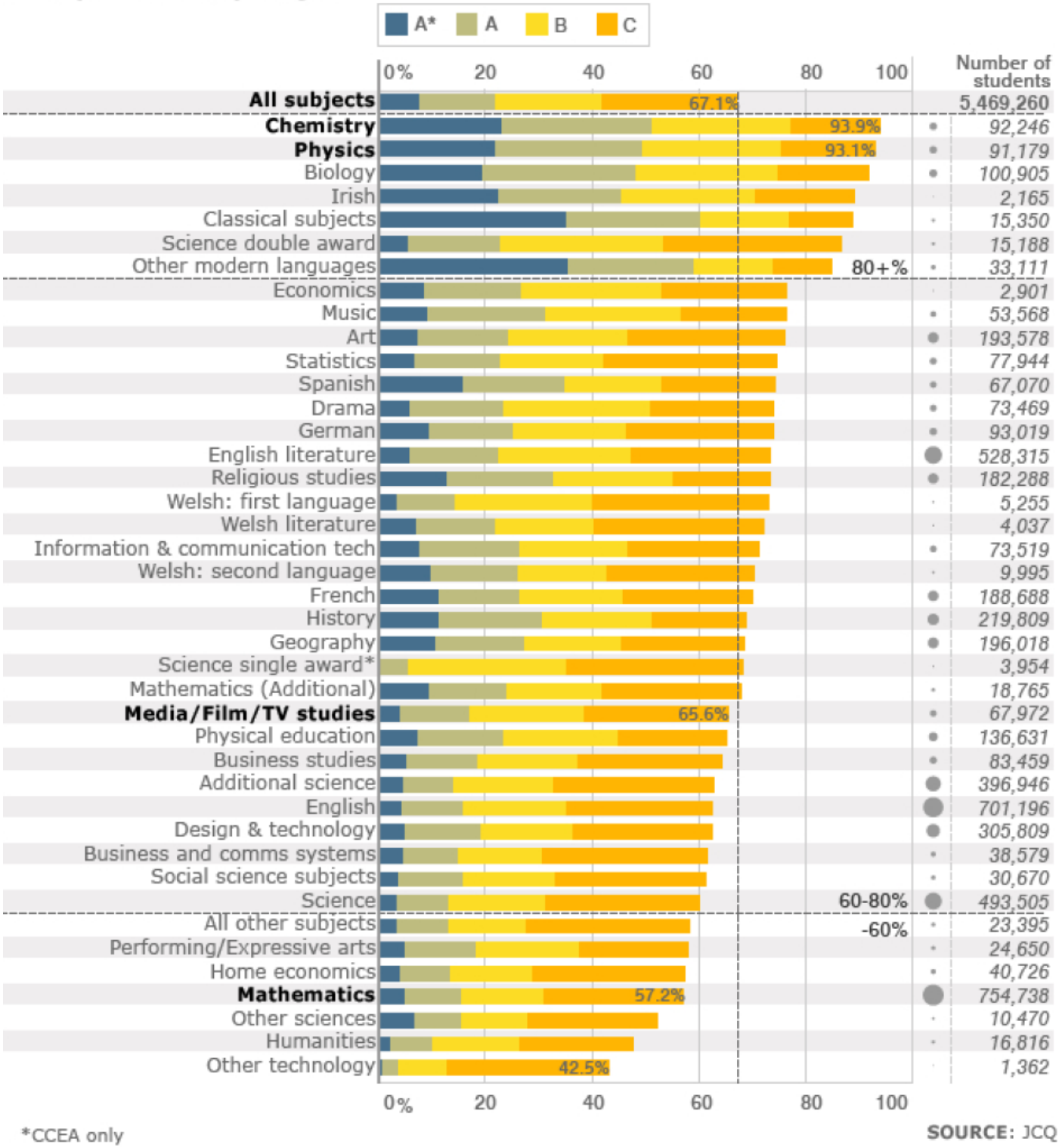
(c) Colin says "the separate sciences are easy because virtually no-one scores less than a C and there are lots of A and A*".

(d) Debbie says "people say media studies is a soft subject, but look! It's way down the table here."

Finally,

(e) Make your own statement about something you notice in the data and give evidence to support what you say.

GCSE pass rates by subject



*CCEA only

SOURCE: JCQ

END OF EXAMINATION